



Nonlinear flexural analysis of composite beam with Inverted-T steel girder and UHPC slab considering partial interaction

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ABSTRACT

The composite beam with inverted-T steel girder and UHPC slab connected by studs arranged in the web of the girder was developed to further the economy in material and best exploit the properties of UHPC. However, the composite beam may experience premature failure due to the occurrence of slip at the shear interface. This so-called partial interaction needs to be addressed in the flexural design for field application of the developed composite beam. The present study proposes a nonlinear analysis method for the evaluation of the flexural behavior of the composite beam considering partial interaction. An analysis method using the Fourier series to approximate the internal forces in the beam considering slip is coupled with a sectional analysis method accounting for inelastic behavior of the steel girder and nonlinear behavior of UHPC. The model considering the slip effect induced by partial interaction is seen to realistically and accurately simulate the actual behavior of the composite beam.

1. Introduction

A steel-concrete composite member interconnects the steel beam with the concrete slab by means of shear connectors or studs to achieve a monolithic structure with higher span-to-depth ratio and higher stiffness ratio than the structure using only one material. The composite beam is designed so that the ultimate limit state of either steel beam or concrete slab due to bending and axial force is reached before the failure of the shear interface [1–3]. However, the beam may fail prematurely due to the occurrence of slip at the shear interface. Partial interaction happens because of the flexibility of the connectors and because of the impossibility to accommodate the number of shear connectors required for achieving complete interaction [2].

Fig. 1 compares the strain distributions in the conventional composite beam section when considering ideal-rigid, partial-flexible and no shear connection [3]. Assuming that the steel beam and concrete slab experience the same small vertical deflection $w(x)$, both members will have the same rotations $w'(x)$. Because of these rotations, the consideration of the flexibility of the shear connection causes different strains between the bottom of the concrete slab and the top of the steel beam. Therefore, slip ($\Delta\epsilon$) occurs in the shear interface and becomes maximum when the stiffness of the connection is ignored (no shear connection).

Fig. 2 compares the bending moment-deflection curves of a simply supported composite beam with rigid, flexible and no shear connection [3]. Partial interaction lets the composite beam behave with complete interaction at early loading but the shear connection can not transfer the shear force under greater loads to finally generate slip at the shear interface. Partial interaction may thus significantly reduce the flexural resistance of the composite beam due to the occurrence of slip at the shear interface.

In view of its influence on the behavior of composite members, partial interaction has been dealt in numerous works. Bärtschi [3] provided a comprehensive summary of the early works done on this topic. Starting with the classic partial interaction solution derived for uniform shear connection in a simply supported beam by Newmark et al. [4], studies attempted to derive exact analytical solutions [5–8]. Ranzi et al. [5] proposed a generic modeling of composite steel-concrete beams with elastic shear connection for various boundary conditions. To avoid the use of shear correction factor, Chakrabarti et al. [6] presented a finite element model based on higher-order beam theory to consider the effect of partial shear interaction between layers and transverse shear deformation of the composite beam. Turmo et al. [7] suggested an intuitive finite element model for the analysis of composite beams with partial interaction using 6 different types of frame elements to deal with any

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combination of loading and boundary conditions. Recently, Zeng et al. [8] constructed a composite beam element based on the Euler-Bernoulli theory and adopted stepwise uniform distribution of the shear connectors to analyze the effect of their layout on the behavior of composite beams with interfacial slip. However, despite of their effectiveness, these analytical procedures remain difficult and complex to apply for practical design work [2].

Other researchers developed more practical approximate procedures to consider the partial interaction in composite beams [9–12]. Based upon the exact solution derived from the linear partial interaction theory, Jasim et al. [9] constructed a design chart to calculate the midspan deflection of simply supported beams with partial shear connection. Seracino et al. [10] developed a simplified mathematical model that uses a magnification factor to relate the partial interaction curvature to the full interaction curvature in composite beams. Girhammar [11] suggested a procedure to predict the deflections and internal actions and stresses by replacing the ideal composite bending stiffness with the partially composite bending stiffness. Kim et al. [12] adopted the Fourier series to approximate the internal axial forces of a simply supported composite beam with partial interaction. Park et al. [13] extended this method to approximate the internal axial forces and deflections of composite beams with partial interaction under arbitrary boundary and loading conditions.

On the other hand, the composite beam with inverted-T steel girder and UHPC slab was developed to further the economy in material and exploit fully the properties of UHPC. In this configuration, the studs must be disposed in the web of the girder for the composition of the steel girder with the UHPC slab (Fig. 3). Based upon a series of test on eight full-scale members, this previous study proposed an analysis method accounting the tension-softening behavior of UHPC in the material model to finally recommended stud spacing ranging between 100 mm and 2 to 3 times the thickness of the slab [14]. Paying attention to the absence of consensus on the limit compressive strength of UHPC, a subsequent study improved the efficiency of the composite beam. This study concluded that the composite beam could develop sufficient ductile behavior with respect to the 6-mm slip limit of Eurocode-4 regardless of the compressive strength of UHPC, steel fiber content, stud spacing and slab thickness [15].

In these previous studies [14,15] by our research team, ideal rigid shear connection was assumed in the flexural design of the composite beam. With regards to the way by which the concrete slab and steel beam are connected, the integrity of the structure relies primarily on the interfacial interaction provided by the shear connection since it would be too risky to count on the frictional resistance. However, achieving perfectly rigid shear connection is realistically impossible. Therefore, it is necessary to consider the partial interaction of the shear connection for the proposed composite beam in the most practical manner to help the designer conduct more audacious but safe design. The present paper proposes a nonlinear analytic model for the evaluation of the flexural behavior of the composite beam with inverted-T steel girder and UHPC slab considering partial interaction. This new model couples the approximate analysis method using the Fourier series [13], based on the

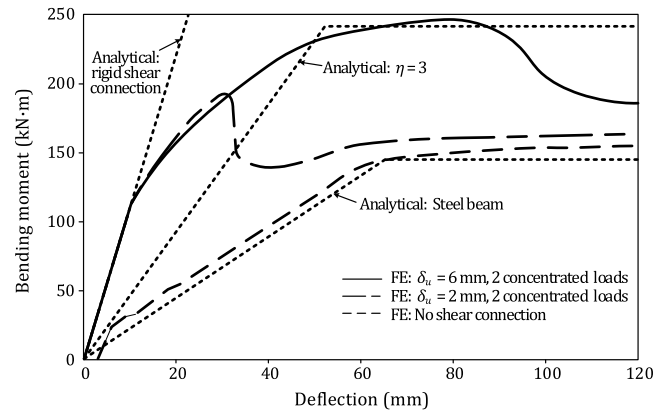


Fig. 2. Load-deflection behavior of simply supported composite beam with partial connection ($L = 7.2$ m, steel section IPE 270, $f_y = 300$ MPa, degree of partial interaction $\eta = 0.3$, δ_u = deformation capacity of shear connection) [3].

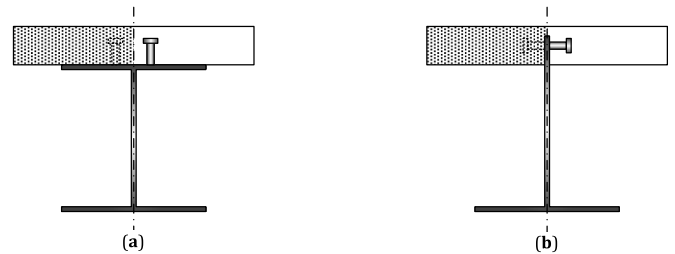


Fig. 3. Shear connection configuration: (a) Conventional composite beam; (b) Composite beam with inverted-T steel girder and UHPC slab [14,15].

linear interaction analysis and without consideration of the inelastic behavior of concrete, with the sectional layered analysis [14,15] to account not only the partial interaction but also the inelastic behavior of the steel girder as well as the tensile behavior and nonlinear behavior of UHPC. The proposed model is validated by comparison with the experimental results obtained previously.

1.1. Proposed nonlinear analytic model for composite beam with partial interaction

The following gives a brief summary of the analysis method approximating the internal forces and deflections of composite beams with partial interaction based on the linear interaction analysis [13], the sectional analysis considering the behavior of the materials [14] and the method by which these two concepts are coupled to construct the proposed nonlinear analytic model. Each previous method is explained in detail in Refs. [13,14].

Fig. 4 depicts the cross section of the composite beam with inverted-T steel girder and UHPC slab together with its longitudinal differential

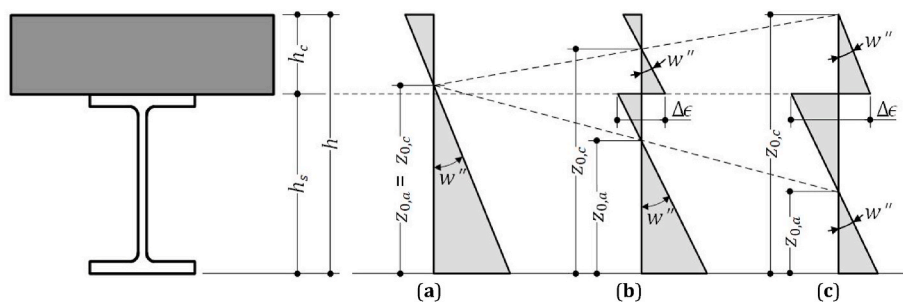


Fig. 1. Strain distribution in composite beam section: (a) Ideal-rigid; (b) Partial-flexible; (c) No shear connection [3].

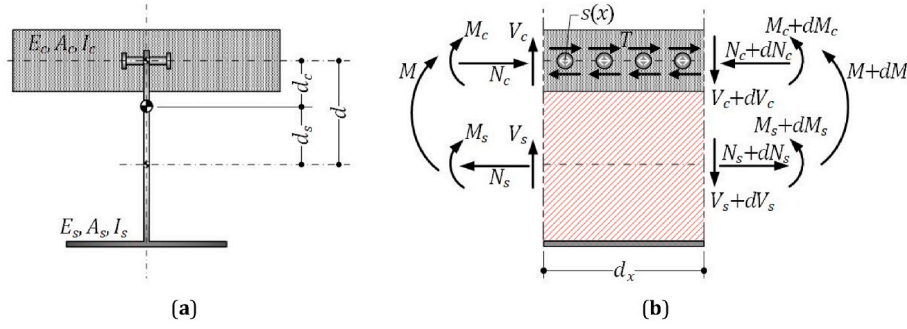


Fig. 4. (a) Cross section of composite beam with inverted-T steel girder and UHPC slab; (b) Longitudinal differential element of composite beam.

element of length d_x . In Fig. 4, $N_c(x)$ and $N_s(x)$ are the internal axial forces of concrete and steel, respectively; $M_c(x)$ and $M_s(x)$ are the internal moments of concrete and steel, respectively; and, d is distance between the centroids of the steel beam and the concrete slab.

Based on the linear elastic partial interaction theory developed by Newmark et al. [4], the internal axial forces and moments can be related by the equilibrium of the differential composite beam element shown in Fig. 4 under the assumption that the steel beam and the concrete slab have the same rotation and curvature.

$$N(x) = N_c(x) = -N_s(x), \quad (1)$$

$$M(x) = M_c(x) + M_s(x) + N(x) \cdot d \quad (2)$$

Assuming linear-elastic load-slip behavior and continuous shear connection along the steel-concrete interface [4], the longitudinal shear flow $T(x)$, the internal axial force $N(x)$ and the slip $s(x)$ can be related linearly as follows by the spring constant of interfacial layer, K .

$$T(x) = \frac{dN(x)}{dx} = K \cdot s(x), \quad (3)$$

The differential equation relating these internal forces can be obtained from the equilibrium and compatibility conditions [11].

$$\frac{d^2 N(x)}{dx^2} - \alpha^2 N(x) = -\beta M(x) \quad (4)$$

with

$$\alpha^2 = K \left(\frac{1}{E_s A_s} + \frac{1}{E_c A_c} + \frac{d^2}{E_s I_s + E_c I_c} \right), \quad (5)$$

$$\beta = \frac{Kd}{E_s I_s + E_c I_c}. \quad (6)$$

Eq. (4) implies to find the minimum (stationary) value of the definite integral, $I[N(x)]$ with boundary conditions $N(a)$ and $N(b)$ expressed in Eq. (7), which satisfies the Euler-Lagrange equation in Eq. (8).

$$I[N(x)] = \int_a^b F(x, N, N', N'') dx \quad (7)$$

$$\frac{\partial F(x, N, N', N'')}{\partial N} - \frac{d}{dx} \left(\frac{\partial F(x, N, N', N'')}{\partial N'} \right) + \frac{d^2}{dx^2} \left(\frac{\partial F(x, N, N', N'')}{\partial N''} \right) = 0 \quad (8)$$

Given

$$F[N(x)] = N(x) \left[\frac{1}{2} \left(\frac{d^2 N(x)}{dx^2} - \alpha^2 N(x) \right) + \beta M(x) \right], \quad (9)$$

the partial derivatives of Eq. (9) are

$$\frac{\partial F(x, N, N', N'')}{\partial N} = \frac{1}{2} \left(\frac{d^2 N(x)}{dx^2} - \alpha^2 N(x) \right) + \beta M(x) \quad (10)$$

$$\frac{\partial F(x, N, N', N'')}{\partial N'} = 0, \quad (11)$$

$$\frac{\partial F(x, N, N', N'')}{\partial N''} = \frac{1}{2} N(x). \quad (12)$$

Substituting Eq. (9) in Eq. (7) gives

$$I[N(x)] = \int_0^L N(x) \left[\frac{1}{2} \left(\frac{d^2 N(x)}{dx^2} - \alpha^2 N(x) \right) + \beta M(x) \right] dx. \quad (13)$$

An approximate solution of $N(x)$ in Eq. (13) can be obtained using the Fourier series of order n ,

$$N(x) = \sum_{i=1}^n a_i p_i(x), \quad (14)$$

where a_i are unknown real coefficients; and, $p_i(x)$ depend on the boundary conditions.

Substituting Eq. (14) in Eq. (13) leads to

$$I[a_1, a_2, \dots, a_n] = - \sum_{j=1}^n \sum_{i=1}^n a_i a_j \frac{1}{2} \int_0^L p_i'(x) p_j'(x) dx - \alpha^2 \sum_{j=1}^n a_j \int_0^L p_j(x) dx \\ \times \sum_{i=1}^n a_i a_j \frac{1}{2} \int_0^L p_i(x) p_j(x) dx + \beta \sum_{i=1}^n a_i \int_0^L M(x) p_i(x) dx \quad (15)$$

of which minimum can be obtained by stating that its partial derivatives with respect to a_i are zero. These n partial derivatives provide the following formula for determining the coefficients a_i with $i = 1, 2, \dots, n$ [12].

$$\left[\int_0^L p_i'(x) p_j'(x) dx + \alpha^2 \int_0^L p_i(x) p_j(x) dx \right] a_i = \beta \int_0^L M(x) p_j(x) dx \quad (16)$$

For a given set of boundary conditions, the coefficients a_i are obtained and can be used to compute the deflection and internal forces at any position in the composite beam as follows.

$$\frac{d^2 y}{dx^2} = \frac{M(x) - N(x)d}{E_s I_s + E_c I_c}, \quad (17)$$

$$M_k(x) = \frac{E_k I_k}{EI_0} (M(x) + N(x)d), \quad (18)$$

$$V_k(x) = \frac{dM_k(x)}{dx} + V_s(x) d_k. \quad (19)$$

where $k = s$ (steel) or c (concrete); EI_0 = flexural rigidity of non-composite beam section; and, d_k = distance between centroid of k -element (steel beam or concrete slab) and centroid of composite beam.

The internal forces for the simply supported beam (length L) sub-

jected to one concentrated load obtained by using this method are given in Table 1.

The validation of the approximate analysis using the Fourier series was performed through comparison with the solutions provided by the method of Girhammar [11]. The comparison showed that the errors in the approximate deflection of the simply supported composite beam were less than 0.4% and caused by the truncation error generated when using Eq. (14) with the Fourier series calculated iteratively to the ninth term [12]. Moreover, the maximum relative error ran around 5% in the third term and around 0.5% in the fifth term of the Fourier series regardless of boundary conditions and loading types. The validation revealed that the use of the Fourier series to minimum fifth terms provided an approximate solution with sufficient accuracy [12,13].

However, the results of the previous study [12] by our research team were obtained through an analysis that considered the interaction of the shear connection in elastic state. In view of the materials involved in the developed composite beam, the analysis shall consider the nonlinear behavior of concrete and steel. Consequently, the present study suggests an approximate analysis method for the composite beam with inverted-T steel girder and UHPC slab considering not only the interaction of the shear connection but also the inelastic behavior of the steel girder as well as the ductile and nonlinear behaviors of UHPC. The following presents the analysis method proposed in this study.

For the multi-layered cross-section model in Fig. 5, the strain and neutral axis of concrete and steel of the composite beam are first obtained for the given curvature using the strain compatibility condition and considering the strain distribution in the section. Details about the ductile behavior of UHPC can be found in a previous study by our research team [13].

The following 4-step process starts by assuming an initial curvature φ_0 of the simply supported composite beam, let say 10^{-6} .

- (1) The strains at the top and bottom of the beam section are expressed as follows by means of the strain compatibility conditions.

$$\begin{aligned}\varepsilon_{\text{top}} &= c\varphi, \\ \varepsilon_{\text{bottom}} &= (h - c)\varphi\end{aligned}\quad (20)$$

where φ = curvature; and, h = height of section.

After having assumed the strain distribution, the stress in each layer is computed based upon the stress-strain relations of the steel fiber reinforced concrete and steel reinforcement established in the material modeling. Then, the section forces can be calculated at each layer considering that the resultants of the sectional forces in all the layers are in equilibrium.

$$\begin{aligned}C_c + T_c + T_s &= 0 \\ \int_{A_c} \sigma_c dA_c + \int_{A_s} \sigma_s dA_s &= 0\end{aligned}\quad (21)$$

Table 1

Equations for calculation of Fourier coefficients in case of simply supported composite beam with one concentrated load [12,13].

$N(x) = \sum_{i=1}^n \int_0^L p_i p_i' dx$	$\int_0^L p_i p_j dx$	$M(x)$	$\int_0^L M(x) p_j(x) dx$
$a_i p_i(x)$			
$\sum_{i=1}^n a_i \sin \frac{i\pi x}{L}$	$\begin{cases} \left(\frac{i\pi}{L}\right)^2 \frac{L}{2} \quad (i=j) \\ 0 \quad (i \neq j) \end{cases}$	$\begin{cases} \frac{L}{2} \quad (i=j) \\ 0 \quad (i \neq j) \end{cases}$	1 x x^2 x^3
			$\frac{L}{i\pi} (1 - \cos i\pi)$ $\left(\frac{L}{i\pi}\right)^2 (-i\pi \cos i\pi)$ $\left(\frac{L}{i\pi}\right)^3 [-2 + (2 - i^2 \pi^2) \cos i\pi]$ $\left(\frac{L}{i\pi}\right)^4 [i\pi(6 - i^2 \pi^2) \cos i\pi]$

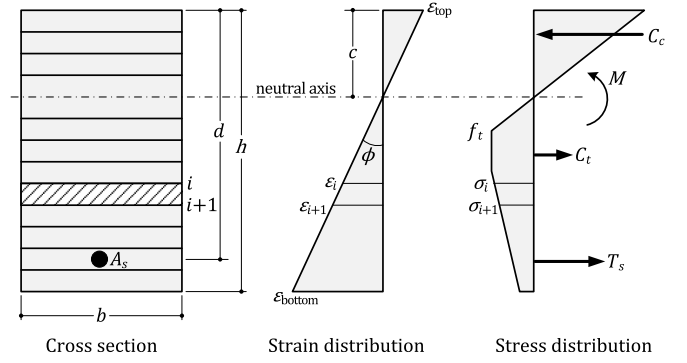


Fig. 5. Distribution of strain and stress in multi-layered section [14].

where C_c, T_c = compressive force and tensile force sustain by concrete, respectively; T_s = tensile force sustained by steel; σ_c = stress of concrete; and, σ_s = stress of steel.

The sectional force, that is the moment, can be calculated based on the stress distribution in the cross-section satisfying the equilibrium.

$$M = \frac{PL}{4} = \int_{A_c} \sigma_c y dA_c + \int_{A_s} \sigma_s y dA_s \quad (22)$$

where P is the load acting on the member of span length L .

- (2) If Eq. (21) is satisfied, the concentrated load P acting at mid-span of the composite beam is obtained by Eq. (22) for the given curvature, φ . And, the positions of the layers at which yield of the steel girder and cracking of concrete occur are also computed.
- (3) The moment of inertia of steel (I_s) and concrete (I_c) are calculated considering the positions of the steel yield layer and concrete cracking layer and are used to compute the values of α^2 and β in Eqs. (5) and (6). The slip $s(x)$ is obtained using Eq. (3) and Eqs. (14)–(19). The deflection δ of the composite beam under the load P obtained in (2) is calculated by integrating Eq. (17). This deflection accounts both contributions of the elastic deflection and the deflection caused by the slip induced by partial interaction. Moreover, the inelastic behavior of the steel girder is considered by excluding the moment of inertia of the yielded part of the steel girder from the calculation.
- (4) The radius of curvature, ρ , considering the deflection obtained in (3) can be determined by Eq. (25) obtained by eliminating θ from the geometric relations of the simply supported beam in Eqs. (23) and (24). And, the strains in steel and concrete are calculated by Eq. (20).

$$\rho = \frac{\delta}{1 - \cos \theta} \quad (23)$$

$$\sin \theta = \frac{L/2}{\rho} \quad (24)$$

$$\rho = \frac{1}{\varphi} = \frac{4\delta^2 + L^2}{8\delta} \quad (25)$$

The process is repeated by increasing the concrete strain by a small increment $\Delta \varepsilon$ until steel or concrete reaches the ultimate strain. Fig. 6 shows the iterative flowchart of the proposed nonlinear analysis method for the composite beam considering both partial interaction at the shear interface and behaviors of the materials. The rigid shear connection can be reproduced by discarding steps (3) and (4) in the process or by setting the spring constant of interfacial layer, K , to the maximum value (considering the entire steel-concrete contact surface). Moreover, the case of no shear connection can be simulated by setting $K = 0$ in Eq. (3).

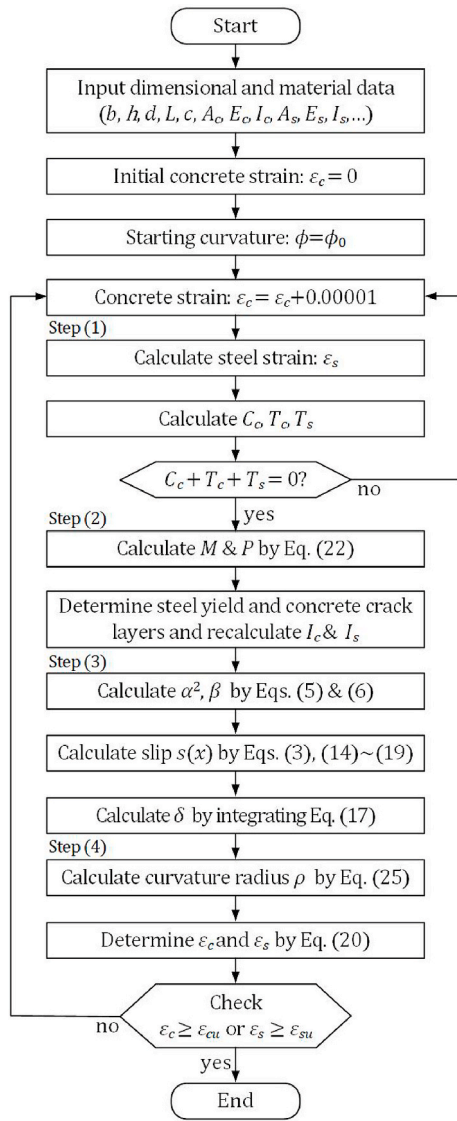


Fig. 6. Flowchart of proposed analysis method for composite beam with partial interaction and considering material behaviors.

2. Validation of proposed method

2.1. Experimental results

The composite beam with inverted-T steel girder and UHPC slab was proposed to further the economy in material and to exploit fully the remarkable strength of UHPC (Fig. 3(b)) [14,15]. The following summarizes the results of tests performed previously by our research team [14] to evaluate the flexural behavior of the composite beam. The experimental results are used to validate the analytic model presented in the previous section.

Eight test members with span length of 2,200 mm were fabricated considering the longitudinal spacing of the studs (50, 100, 200, 400 mm) and slab thickness (50, 100 mm) as test variables (Fig. 7) [14]. The designation of the test members is listed in Table 2. For the steel girder, the bottom flange has thickness of 13 mm and width of 225 mm, and the web is 8-mm thick and 177-mm high. The UHPC slab has thickness of 50 mm and 100 mm and width of 248 mm at 137 mm along the height of the web.

The shear connectors present shank diameter of 13 mm, head diameter of 22 mm, and length of 50 mm. From direct tensile test, the

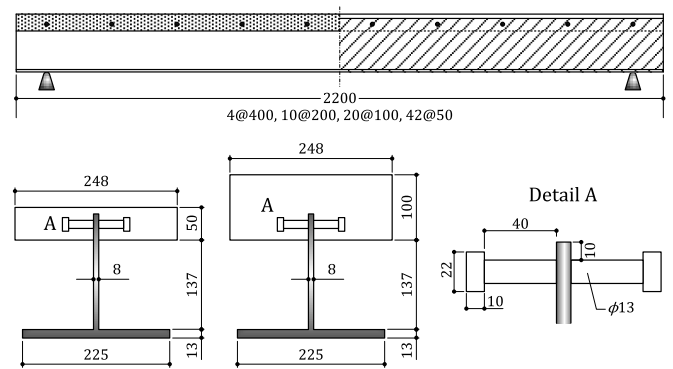


Fig. 7. Shape and dimensions of composite beam specimens for 3-point flexural test [14].

Table 2

Designation and characteristics of composite beam specimens [14].

Designation	Slab thickness (mm)	Stud spacing (mm)
50-50	50	50
50-100	50	100
50-200	50	200
50-400	50	400
100-50	100	50
100-100	100	100
100-200	100	200
100-400	100	400

yield strength and ultimate strength were found to be 370 MPa and 470 MPa, respectively. Structural steel SM490 with yield strength of 396 MPa and ultimate strength of 554 MPa was used for the steel girder.

The mix composition of UHPC adopted for the slab is listed in Table 3 with reference to the Design Recommendations for Ultra-High Performance Concrete K-UHPC [16]. Note that the proportions are indicated in weight ratio by setting the amount of cement to 1. Steel fibers (length of 13 mm) with two different diameters of 19.5 mm and 16.3 mm were admixed in 2% volume fraction at ratio of 2:1, respectively. The density of the steel fiber is 7,500 kg/m³ and the yield strength is 2,500 MPa. In Table 3, Zr, E.A., S.R.A., A.F.A. indicate zirconium, expansive agent, shrinkage reducing agent and antifoaming agent, respectively.

In the compressive strength test, the UHPC cylinders made with the mix proportions of Table 2 behaved linearly until their sudden failure at the ultimate strain (ε_{cu}) in agreement with the well-known linear behavior of steel fiber reinforced UHPC reported in the literature [18, 19]. The tests gave average values of 183 MPa for the compressive strength, 45 GPa for the elastic modulus and 0.00406 for the ultimate strain. The average tensile strength was found to be 11.7 MPa, which is significantly higher than that observed in normal concrete.

Three-point loading test was conducted on the specimens. A total of 18 strain gages with 9 concrete strain gages and 9 steel strain gages were attached to the front and rear faces of the specimens and, two LVDTs (linear variable differential transducer) were installed at 1/2 and 1/3 of the span to measure the deflection. Strain gages were also attached to the top and side faces of all the studs located inside the 400-mm wide area around the mid-span. Table 4 arranges the results of the loading test. The relative slip at 90% of the failure load and the characteristic relative slip extracted from the measurements in the tests are also indicated. Note that for excessively large stud spacing, Eurocode-4 [17] defines the characteristic relative slip, δ_{uk}, as 0.9δ_u, where δ_u is the relative slip measured at load corresponding to the ultimate load reduced by 10%. Eurocode-4 assesses ductile behavior when this characteristic relative slip is larger than 6 mm and states that stable behavior is achieved by the composite member in such case. The test results reveal that the test members developed sufficient ductile behavior.

Table 3

Mix proportions of UHPC used for concrete slab of composite beam [14].

Cement	Zr	Sand	Filler	E.A.	S.R.A.	Plasticizer	Water	A.F.A.	Steel fiber
1	0.25	1.10	0.30	0.075	0.01	0.03	0.23	0.01	0.10 (19.5 mm) 0.05 (16.3 mm)

Table 4

Crack, yield, ultimate, failure load and characteristic slip measured in loading test of composite beam specimens [14].

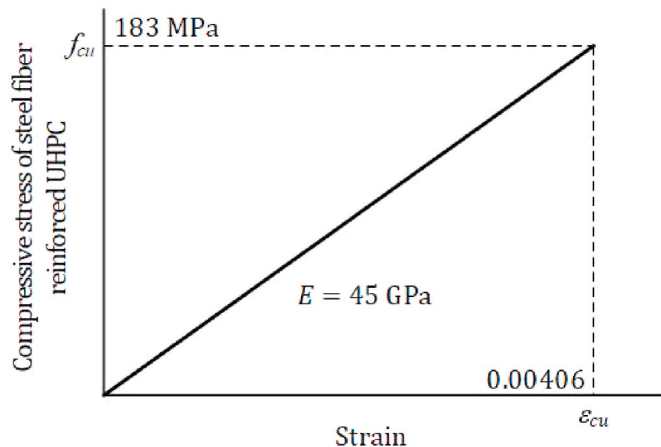
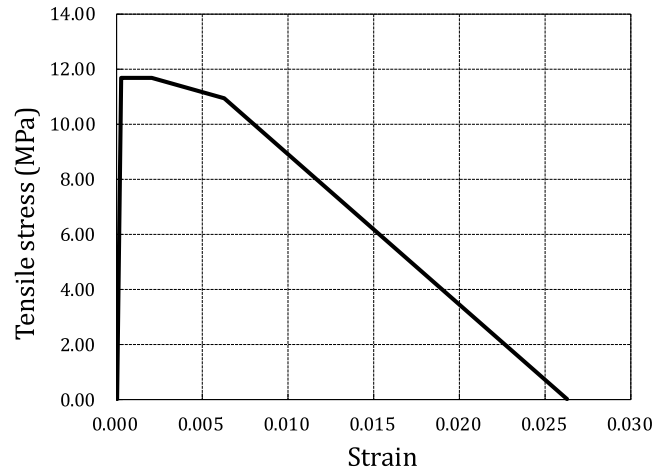
Test member	Crack load (kN)	Yield load, P_y (kN)	Ultimate load, P_u (kN)	Failure load (kN)	δ_u (mm)	δ_{uk} (mm)
50-50	–	428	573	554	11.72	10.54
50-100	–	431	551	523	11.70	10.53
50-200	414	409	451	425	10.87	9.78
50-400	146	279	281	277	16.84	15.16
100-50	731	567	807	806	12.28	11.05
100-100	642	559	756	752	11.70	10.53
100-200	476	536	583	563	10.77	9.69
100-400	308	–	435	426	10.45	9.41

2.2. Material modeling

For the analyses, bilinear model is adopted for steel and the models for the compressive and tensile behaviors of UHPC are shown in Figs. 8–9. The 3-point bending test on notched beams proposed by RILEM [20] and JCI [21] known to be particularly adapted for steel fiber reinforced concrete was applied to describe the tension softening behavior. The tension softening model was obtained analytically based on the fictitious crack model of Hillerborg et al. [22] and is represented as a multi-linear curve. The slopes of the crack mouth opening displacement (CMOD) and tension softening curve were determined by optimization of the load obtained from the experimental load and crack equation. At each analysis step, the constitutive equations are formulated for every CMOD to compute the optimal tension softening curve [14].

3. Comparison of analytic and experimental results

The analysis method presented in Section 2 was applied by iterative calculation of the Fourier series to the ninth term. In the following, the results designated “rigid stud” refer to those ignoring the slip effect induced by partial shear interaction and obtained previously by our research team. The analytic and experimental results are compared in Table 5 and drawn in Figs. 10–18 in the following sections.

**Fig. 8.** Compressive stress-strain model of steel fiber reinforced UHPC [14].**Fig. 9.** Tensile stress-strain model of steel fiber reinforced UHPC [14].

3.1. Load-deflection relation

Figs. 10–11 compare the experimental load-deflection curves with the analytic curves considering rigid shear connection and the analytic curves considering partial interaction by the analysis method proposed in this study for the members presented in Table 2. For the members with stud spacing of 50 and 100 mm, the load-deflection curves obtained by the proposed analysis considering the slip induced by partial interaction are very similar to those obtained by the analysis considering rigid shear connection. For the members with wider stud spacing of 200 and 400 mm, the load-deflection curves obtained by the proposed partial interaction analysis are closer to the experimental curves. This difference according to the stud spacing can be explained by the monolithic behavior of the composite beam with shorter stud spacing whereas the slip becomes more pronounced with wider stud spacing, which results in earlier damage of the shear connection and some loss in the flexural performance of the composite beam. Consequently, the proposed analysis model considering partial interaction simulates appropriately the behavior of the composite beam. Additionally, member 50-400 with slab thickness of 50 mm and stud spacing of 400 mm exhibits distinctively poor structural performance.

3.2. Load-concrete strain relation

Figs. 12–13 compare the experimental load-concrete strain curves with the analytic curves considering rigid shear connection and the analytic curves considering partial interaction by the analysis method proposed in this study. Here, the concrete strain is the one at the upper extreme fiber of the composite section. Observations resembling those discussed for the load-displacement curves can be done. Specifically, the composite beam members with shorter stud spacing behave monolithically due to the smaller occurrence of slip while the members with larger stud spacing exhibit poorer structural performance due to earlier damage of the studs caused by the greater slip induced by partial interaction. The proposed analysis model appears thus to simulate appropriately the behavior of the composite beam in term of the concrete strain.

Moreover, the peak strain and ultimate strain of concrete resulting

Table 5

Comparison of experimental (E) and analytic (A) displacements (δ), upper extreme fiber concrete strains (ϵ_c), lower extreme fiber steel strains (ϵ_s), internal axial forces (N) and maximum slip lengths (s) for considered test members.

Member		Ultimate						Failure			
		Load (kN)	δ (mm)	ϵ_c ($\mu\epsilon$)	ϵ_s ($\mu\epsilon$)	N (kN)	s (mm)	Load (kN)	δ (mm)	ϵ_c ($\mu\epsilon$)	ϵ_s ($\mu\epsilon$)
50-rigid stud 50–50	A	517	27.02	−4020	6426	–	–	517	27.02	−4020	6426
	E	573	17.16	−3993	4009	1206	–	554	17.66	−3993	4009
50–100	A	498	18.48	−4020	3370	997	1.258	498	18.48	−4020	3370
	E	551	19.85	−2814	11093	997	–	523	22.01	−2814	11350
50–200	A	498	18.67	−4060	3410	990	1.309	498	18.67	−4060	3410
	E	451	21.72	−3020	2608	564	–	425	24.74	−3255	2747
50–400	A	455	17.74	−3860	3240	884	1.360	455	17.74	−3860	3240
	E	281	29.27	−2856	2061	345	–	277	39.71	−3696	2138
100-rigid stud 100–50	A	280	21.74	−4730	3970	302	3.555	280	21.74	−4730	3970
	A	734	26.02	−4043	8618	–	–	712	26.52	−4442	8552
100–100	E	807	19.46	−4271	10858	1686	–	806	19.61	−4271	11242
	A	695	18.07	−4380	4660	1394	0.833	695	18.07	−4380	4660
100–200	E	756	20.00	−4073	5016	1124	–	752	21.09	−4216	6315
	A	642	17.46	−4230	4500	1289	0.836	642	17.46	−4230	4500
100–400	E	583	17.51	−2213	2092	823	–	563	19.10	−2210	2033
	A	598	16.89	−4090	4350	1189	0.896	598	16.89	−4090	4350
	E	435	25.45	−2117	1809	525	–	426	26.96	−2177	1816
	A	445	15.87	−3840	4090	1021	0.829	445	15.87	−3840	4090

from the analysis considering ideal-rigid shear connection approached 0.0030 and 0.0043, respectively. Besides, the values produced by the proposed analysis considering partial interaction approach 0.0028 and 0.0040, respectively. Compared to the experimental values, the analytic concrete strains appear to be well simulated by the rigid-connection analysis for shorter stud spacing and by the partial interaction analysis for larger stud spacing.

3.3. Load-steel strain relation

Figs. 14–15 compare the experimental load-steel tensile strain curves with the analytic curves considering rigid shear connection and the analytic curves considering partial interaction by the analysis method proposed in this study. The steel strain is the one at the lower extreme fiber of the composite section. Here also, the same discussion as for the load-displacement curves can be done and the proposed analysis model considering the slip induced by partial interaction simulates accurately the behavior of the composite beam in term of the steel girder strain.

In addition, ductile behavior can be clearly distinguished after the yield of steel in the members with thick slab thickness whereas the members with thin slab thickness develop relatively shorter ductile plateau.

3.4. Load-internal axial force relation

Figs. 16–17 compare the experimental load-internal axial force curves with the analytic curves considering partial interaction by the analysis method proposed in this study. Here, the experimental internal axial force was calculated by multiplying the average of the strains at the upper and lower extreme fiber of concrete by the elastic modulus of the concrete section; and, the analytic internal axial force is the solution of Eq. (2). The experimental and analytic results are in very good agreement, which indicate that the approximate solution using the Fourier series reproduces accurately the actual behavior of the test members. Note that at the time of the tests, the concrete and steel strains of some members increased disproportionately when approaching the ultimate state. Such experimental error explains the irregularity of some experimental curves together with the non-consideration of the loss of cross-sectional area and elastic modulus of concrete in the calculation.

3.5. Load-slip relation

Fig. 18 compares the analytic load-maximum slip length curves

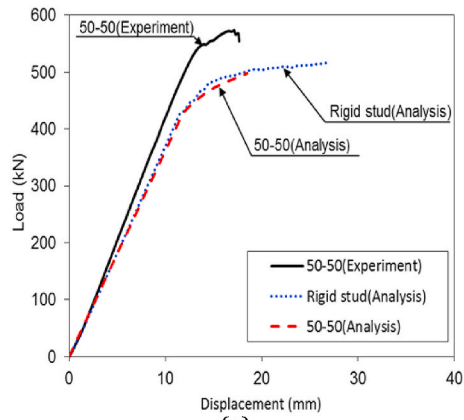
according to the stud spacing of the test members. The comparison is done only for the results obtained by the proposed analysis method considering partial interaction. In addition, the indicated maximum slip length is the one occurring at the support of the composite beam. In view of the graphs, the same behavior is observed in all the specimens with narrow stud spacing or thick slab thickness. However, the maximum slip becomes longer as much as the stud spacing is large and this tendency is more pronounced with thinner slab thickness. Especially, specimen 50–400 experienced very large slip and significantly smaller ultimate load because of the practical impossibility to develop monolithic behavior due to its excessively wide stud spacing and thin slab. Consequently, the composite behavior appears to be impossible due to the negligible role played by the shear connection when the stud spacing is greater than 4 times the slab thickness.

With regards to the ductile behavior limit of 6 mm recommended in Eurocode-4 [17], the developed composite beam with inverted-T steel girder and UHPC slab can satisfy the limit by adopting stud spacing between 100 mm and 2 to 3 times (less than 4 times) the thickness of the slab even when considering partial interaction in the design.

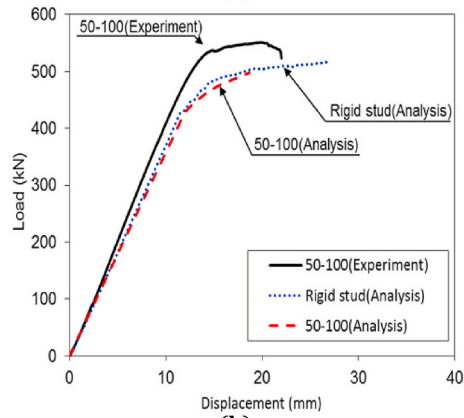
4. Conclusions

The present study proposed a nonlinear analysis model for composite beams considering the slip effect at the shear interface due to partial interaction as well as the behavioral characteristics of the materials involved in the composite beam. The proposed model was applied to analyze the flexural behavior of the composite beam with inverted-T steel girder and UHPC slab developed previously. The test members used to validate the proposed model considered the longitudinal spacing of the studs and slab thickness as test variables. The following conclusions can be drawn.

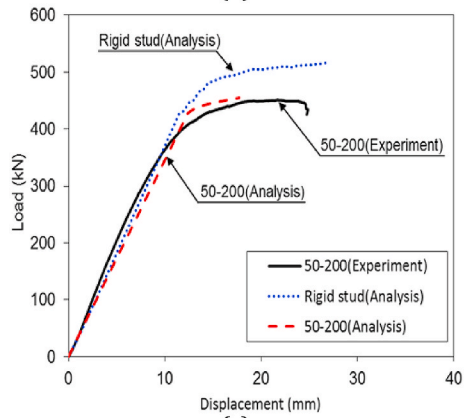
- The composite beam members with shorter stud spacing behaved monolithically due to the smaller occurrence of slip while the members with larger stud spacing exhibited poorer structural performance due to earlier damage of the studs caused by the greater slip induced by the partial interaction.
- In term of the load-deflection and load-concrete strain relations, the analysis assuming ideal-rigid shear connection reproduced sufficiently the behavior of the composite beam for densely arranged studs whereas the analysis considering partial interaction provided accurate simulation for the composite beam with wide stud spacing.



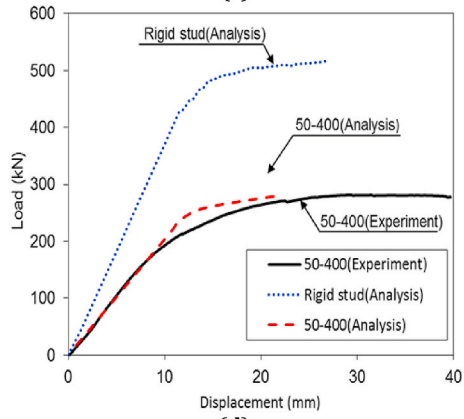
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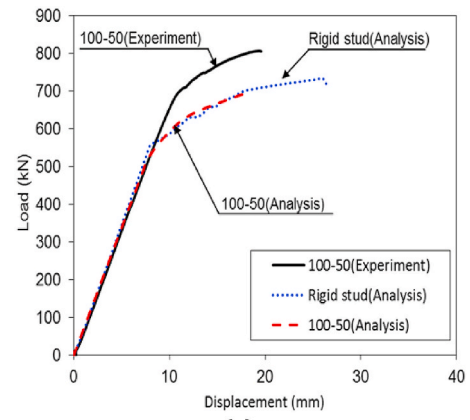
(b)



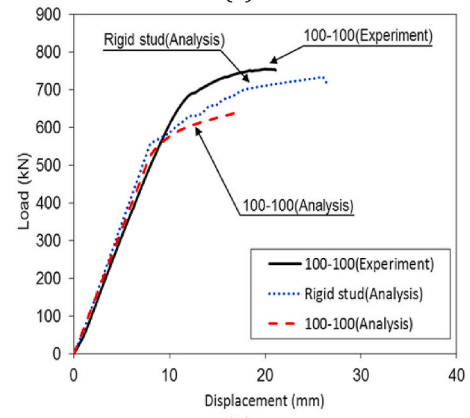
(c)



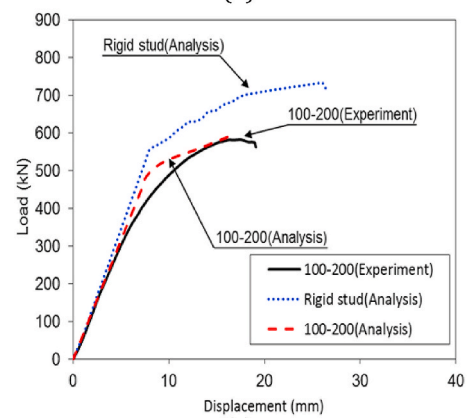
(d)



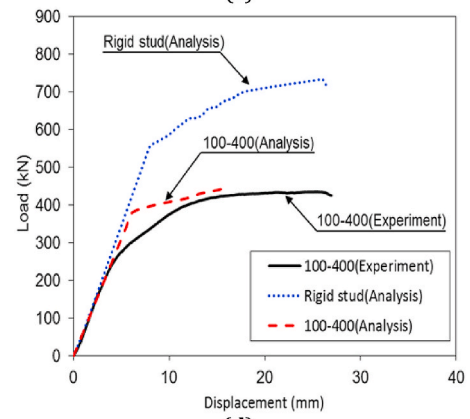
(a)



(b)



(c)



(d)

Fig. 10. Comparison of analytic and experimental load-displacement curves: (a) 50-50; (b) 50-100; (c) 50-200; (d) 50-400.

Fig. 11. Comparison of analytic and experimental load-displacement curves: (a) 100-50; (b) 100-100; (c) 100-200; (d) 100-400.

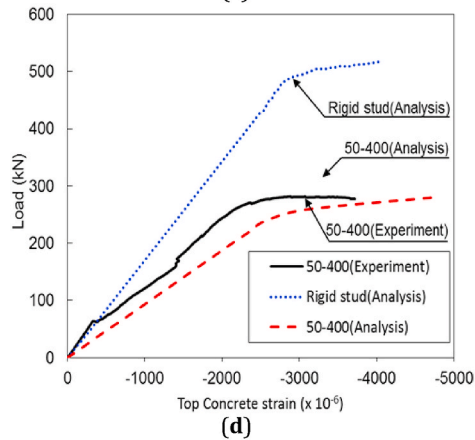
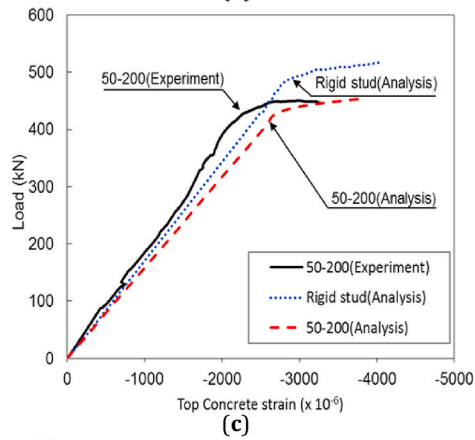
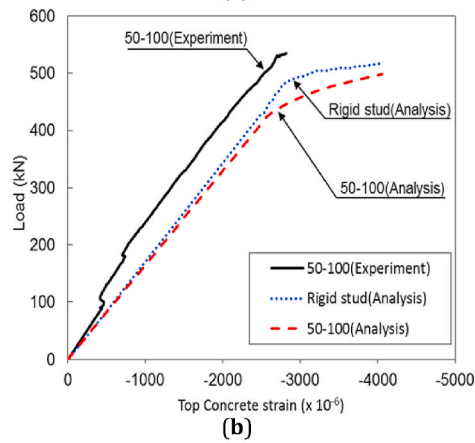
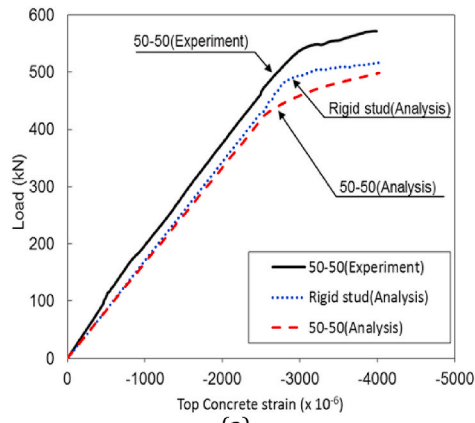


Fig. 12. Comparison of analytic and experimental load-upper extreme fiber concrete strain curves: (a) 50-50; (b) 50-100; (c) 50-200; (d) 50-400.

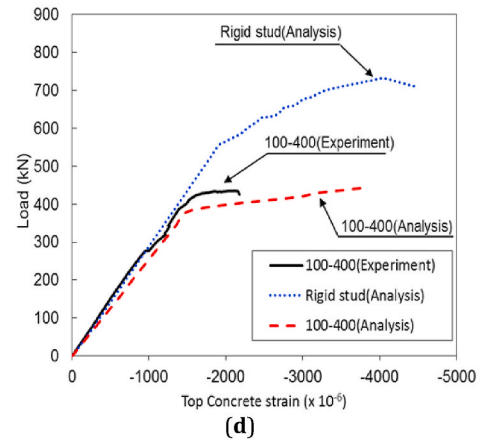
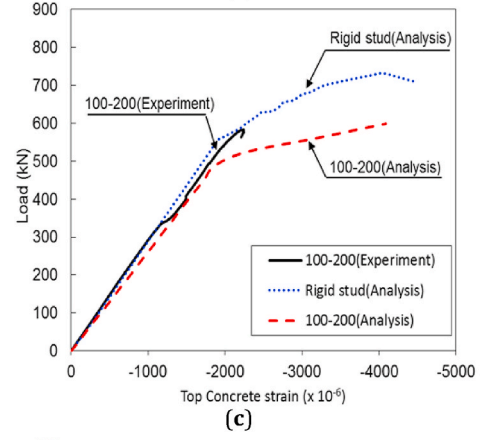
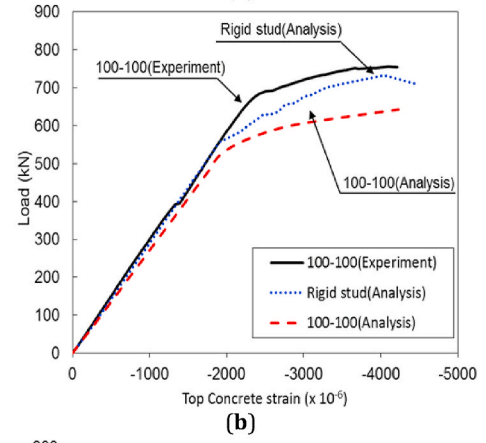
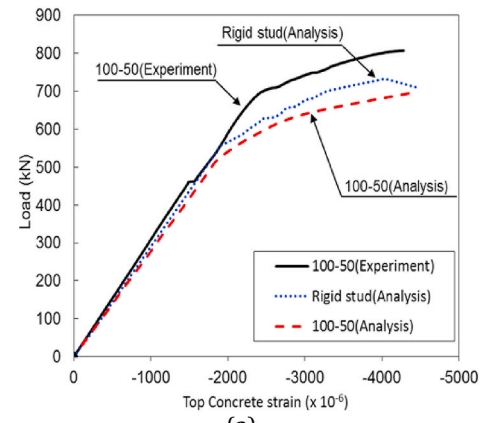
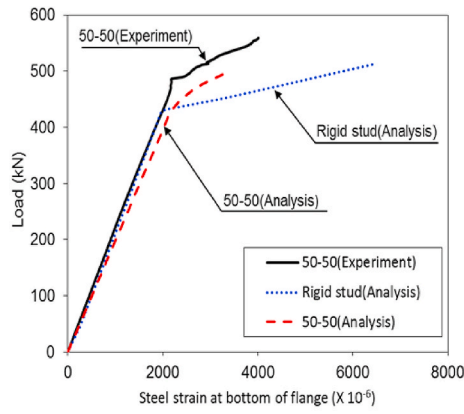
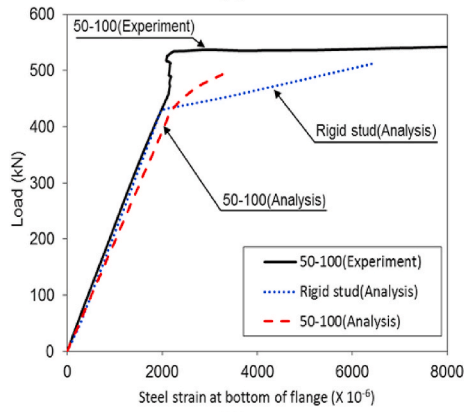


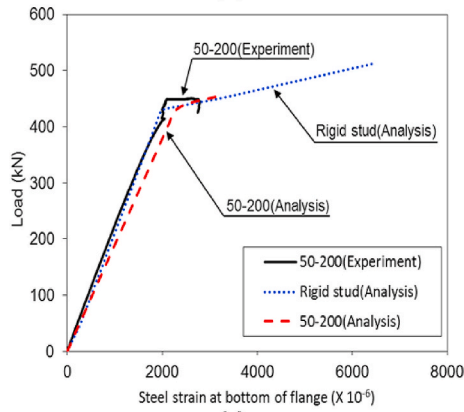
Fig. 13. Comparison of analytic and experimental load-upper extreme fiber concrete strain curves: (a) 100-50; (b) 100-100; (c) 100-200; (d) 100-400.



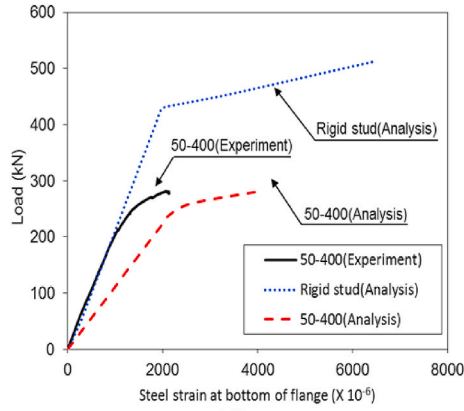
(a)



(b)

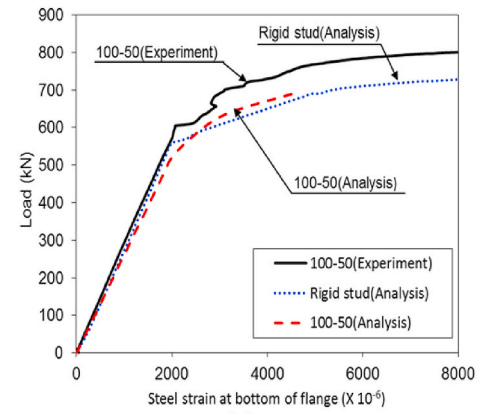


(c)

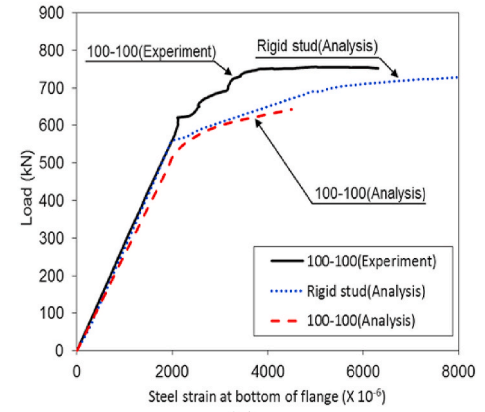


(d)

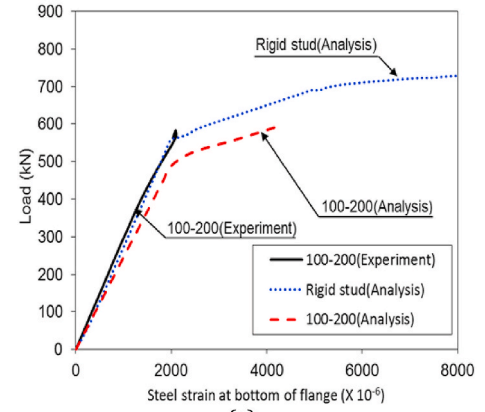
Fig. 14. Comparison of analytic and experimental load-lower extreme fiber steel strain curves: (a) 50-50; (b) 50-100; (c) 50-200; (d) 50-400.



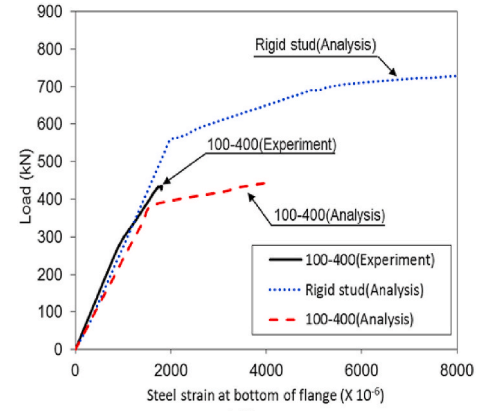
(a)



(b)



(c)



(d)

Fig. 15. Comparison of analytic and experimental load-lower extreme fiber steel strain curves: (a) 100-50; (b) 100-100; (c) 100-200; (d) 100-400.

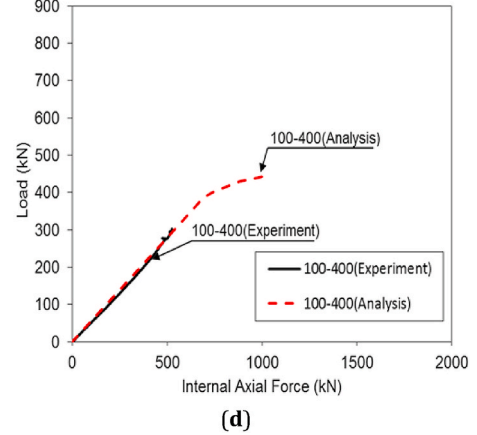
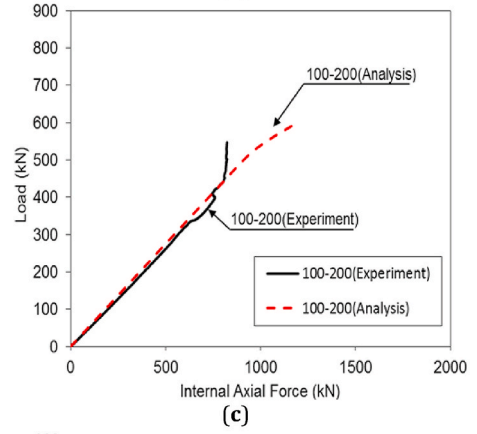
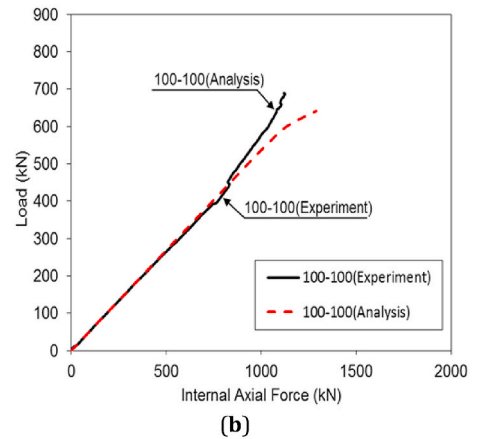
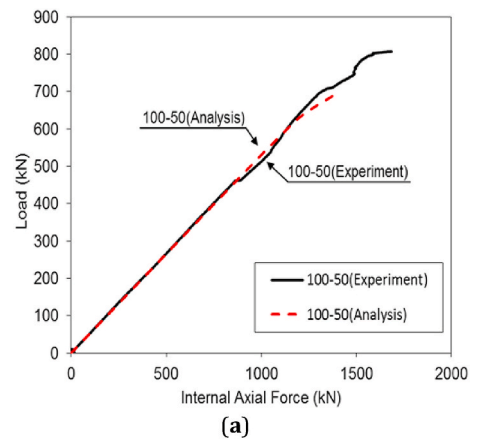
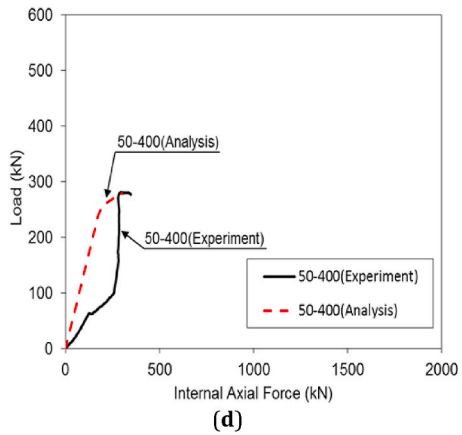
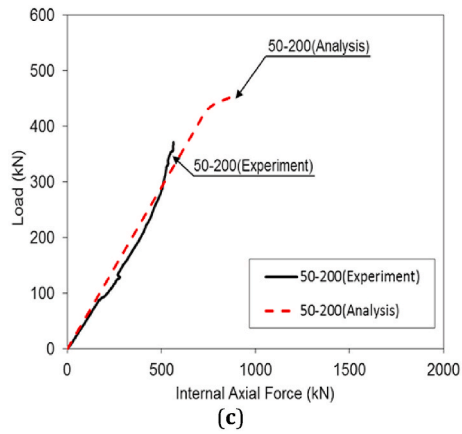
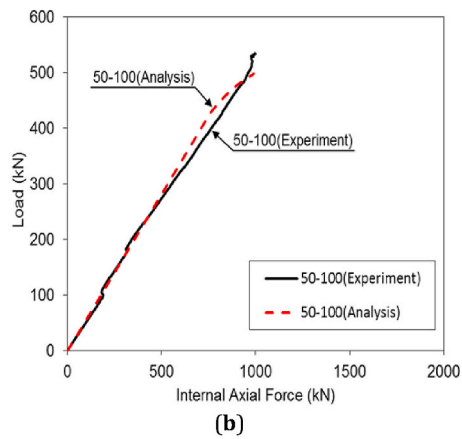
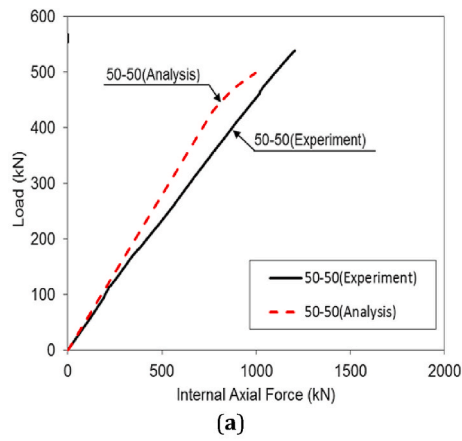


Fig. 16. Comparison of analytic and experimental load-internal axial force curves: (a) 50-50; (b) 50-100; (c) 50-200; (d) 50-400.

Fig. 17. Comparison of analytic and experimental load-internal axial force curves: (a) 100-50; (b) 100-100; (c) 100-200; (d) 100-400.

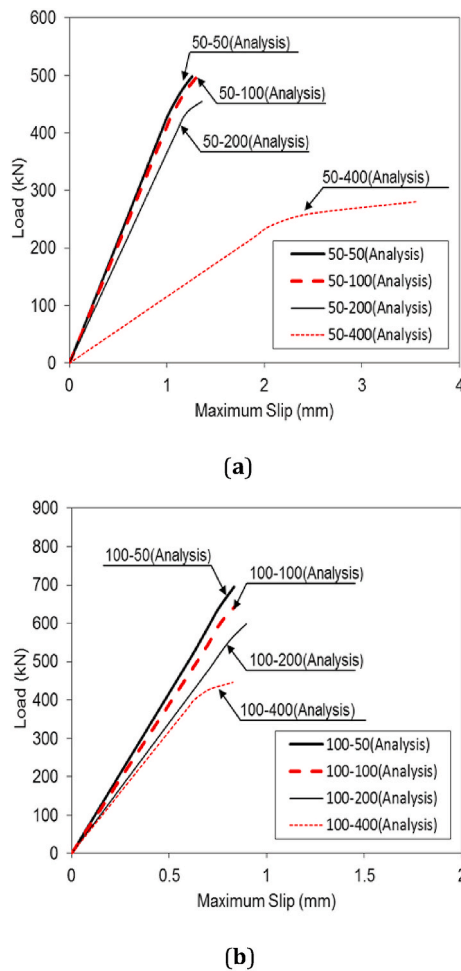


Fig. 18. Analytic load-maximum slip length curves: (a) Slab thickness of 50 mm; (b) Slab thickness of 100 mm.

- From the comparison of the analytic and experimental load-steel strain relations, post-yield ductile behavior could be clearly distinguished in the members with thick slab thickness and the members with thin slab thickness developed relatively shorter ductile plateau.
- The analysis method using the Fourier series to approximate the internal forces in the beam considering the slip was coupled with the sectional analysis method accounting the inelastic behavior of the steel girder and the nonlinear behavior of UHPC. The proposed model considering the slip effect induced by partial interaction simulated realistically and accurately the actual behavior of the composite beam.
- The recommendation to adopt spacing between 100 mm and less than 4 times the thickness of the slab for the composite beam with inverted-T steel girder and UHPC slab satisfied sufficiently the ductile behavior limit of Eurocode-4.

Authors contributions

Conceptualization: Sung-Won Yoo and Jinkyoo F. Choo; Methodology: Sung-Won Yoo; Software: Jun-Hyeok Choi; Validation: Jun-Hyeok Choi and Young-Cheol Choi; Formal analysis: Jun-Hyeok Choi; Investigation: Jun-Hyeok Choi; Resources: Jun-Hyeok Choi and Sung-Won Yoo; Data curation: Sung-Won Yoo; Writing—original draft preparation: Jinkyoo F. Choo and Sung-Won Yoo; Writing—review and editing: Young-Cheol Choi and Jinkyoo F. Choo; Visualization: Jinkyoo F. Choo;

Supervision: Sung-Won Yoo; Project administration: Jinkyoo F. Choo and Sung-Won Yoo; Funding acquisition: Sung-Won Yoo.

Declaration of competing interest

The authors declare that they have no conflict of interest.

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